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# A Study on the Hyperbola

$$y^2 = 87x^2 + 1$$

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## Abstract

The binary quadratic equation $y^2 = 87x^2 + 1$ is considered for obtaining its integral solutions. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration. a few remarkable observations are illustrated.

**Keywords**: Binary quadratic, pell equation , integral solutions, hyperbola , parabola, second order Ramanujan numbers.

## Introduction

Any non-homogeneous binary quadratic equation of the form $y^2 - Dx^2 = 1$, where D is a given positive non-square integer, requiring integer solutions for x and y is called Pellian equation ( also known pell-Fermat equation). In Cartesian co-ordinates, the equation has the form of a hyperbola. The Pellian equation has infinitely many distinct integer solutions as long as D is not a perfect square and the solutions are easily generated recursively from a single fundamental solution, namely the solution with x, y positive integer of smallest possible size. One may refer [1-12] for a few choices of Pellian equations along with their corresponding integer solutions.

The solutions to Pellian equations have long been of interest to mathematicians. Even small values of D can lead to fundamental solution which are quite large. For example, when D=61, the fundamental solution is (1766319049, 226153980). The above results motivated us to search for integer solutions to other choices of Pellian equation. This paper concerns with the pellian equation $y^2 = 87x^2 + 1$, a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration. The process of obtaining second order Ramanujan numbers is illustrated with numerical examples.

**Method of analysis:**

The hyperbola represented by the non-homogeneous quadratic equation under consideration is

$$y^2 = 87x^2 + 1 \quad (1)$$

The smallest positive integer solution is $x_0 = 3, y_0 = 28$

If $(x_n, y_n)$ represents the general solution of(1), then

$$x_n = \left(\frac{1}{2\sqrt{87}}\right) g_n \quad (2)$$

$$y_n = \frac{1}{2} f_n \quad (3)$$

Where,

$$f_n = \left(28 + 3\sqrt{87}\right)^{n+1} + \left(28 - 3\sqrt{87}\right)^{n+1}$$

$$g_n = \left(28 + 3\sqrt{87}\right)^{n+1} - \left(28 - 3\sqrt{87}\right)^{n+1}$$

A few numerical solutions to (1) are presented in table below:

**Table: Numerical solutions**

| **n** | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 3 | 28 |
| 1 | 168 | 1567 |
| 2 | 9405 | 87724 |
| 3 | 52651 | 4910977 |
| 4 | 29475267 | 274926988 |

**Observations:**

- $x_n \equiv 0(\mathrm{mod}\,3), n = 0,1,2......$
- $y_{2n} \equiv 0(\mathrm{mod}\,4), n = 0,1,2......$
- A few interesting relations among the solutions are given below:

- $1567y_{n+2} - y_n - 7308x_{n+2} = 0$
- $x_n - x_{n+2} + 6y_{n+1} = 0$
- $28y_n - 1567y_{n+1} + 261x_{n+2} = 0$
- $28x_{n+1} + 3y_{n+1} - x_{n+2} = 0$
- $28y_{n+2} - y_{n+1} + 261x_{n+2} = 0$
- $x_n - 1567x_{n+2} + 168y_{n+2} = 0$
- $x_{n+1} + 3y_{n+2} - 28x_{n+2} = 0$
- $y_n - 1567y_{n+2} + 14616x_{n+2} = 0$
- $y_{n+1} - 28y_{n+2} - 261x_{n+2} = 0$
- $261x_n - y_{n+1} - 28y_n = 0$
- $261x_{n+1} + y_n - 28y_{n+1} = 0$
- $261x_{n+2} + 28y_n - 1567y_{n+1} = 0$
- $y_{n+2} - 56y_{n+1} + y_n = 0$
- $14616x_n - y_{n+2} + 1567y_n = 0$
- $522x_{n+1} + y_n - y_{n+2} = 0$
- $14616x_{n+2} + y_n - 1567y_{n+2} = 0$
- $56y_{n+1} - y_n - y_{n+2} = 0$
- $261x_n - 28y_{n+2} + 1567y_{n+1} = 0$
- $261x_{n+1} + 28y_{n+1} - y_{n+2} = 0$
- $261x_{n+2} + y_{n+1} + 28y_{n+2} = 0$
- $y_n - 56y_{n+1} + y_{n+2} = 0$

$*$ Expressions representing square integers:

- $(2x_{2n+2} - 56x_{2n+1} + 6)/3$
- $(x_{2n+3} - 1567x_{2n+1} + 168)/84$
- $56y_{2n+2} - 522x_{2n+2} + 2$
- $28(2y_{2n+2} - 536x_{2n+1} + 56)$
- $1567(2y_{2n+3} - 29232x_{2n+1} + 3134)$
- $(56x_{2n+3} - 3134x_{2n+2} + 6)/3$
- $2y_{2n+1} + 2$
- $56y_{2n+2} - 522x_{2n+2} + 2$
- $(29232x_{2n+2} + 56y_{2n+3} + 56)/28$
- $(3134y_{2n+2} - 522x_{2n+3} + 56)/28$
- $3134y_{2n+3} - 29232x_{2n+3} + 2$
- $112y_{2n+2} - 2y_{2n+3} + 2$

*Expressions representing cubical integers:

- $\frac{x_{3n+4} - 1567x_{3n+2}}{84} + 3\left(\frac{x_{n+2} - 1567x_n}{84}\right)$
- $56y_{3n+3} - 522x_{3n+3} + 168y_{n+1} - 1566x_{n+1}$
- $\frac{2y_{3n+3} - 536x_{3n+2} + 3(2y_{n+1} - 536x_n)}{28}$
- $\frac{2y_{3n+4} - 29232x_{3n+2} + 6y_{n+2} - 87696x_n}{1567}$
- $\frac{56x_{3n+4} - 3134x_{3n+3}}{3} + 3\left(\frac{56x_{n+2} - 3134x_{n+1}}{3}\right)$
- $2y_{2n+1} + 3(2y_n)$
- $\frac{-29232x_{3n+3} + 56y_{3n+4}}{28} + 3\left(\frac{-29232x_{n+1} + 56y_{n+2}}{28}\right)$
- $2y_{3n+2} + 6y_n$
- $\frac{3134y_{3n+3} - 522x_{3n+4} + 3402y_{n+1} - 1566x_{n+2}}{28}$
- $3134y_{3n+4} - 29232x_{3n+4} + 3(3134y_{n+2} - 29232x_{n+2})$
- $\frac{336y_{3n+3} - 6y_{3n+4} + 1008y_{n+1} - 18y_{n+2}}{3}$

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Expressions representing biquadratic integers:

- $\frac{2x_{4n+4}-56x_{4n+3}}{3}+4\left(\frac{2x_{n+1}-56x_n}{3}\right)^2-2$
- $\frac{x_{4n+5}-1567x_{4n+3}}{84}+4\left(\frac{x_{n+2}-1567x_n}{84}\right)^2-2$
- $56y_{4n+4}-522x_{4n+4}+4(56y_{n+1}-522x_{n+1})^2-2$
- $\frac{2y_{4n+4}-536x_{4n+3}+8y_{2n+2}-2144x_{2n+1}+168}{28}$
- $\frac{2y_{4n+5}-29232x_{4n+3}}{1567}+4\left(\frac{2y_{2n+3}-29232x_{2n+1}+3134}{1567}\right)-2$
- $\frac{56x_{4n+5}-3134x_{4n+4}}{3}+4\left(\frac{56x_{n+2}-3134x_{n+1}}{3}\right)^2-2$
- $56y_{4n+4}-522x_{4n+4}+4(56y_{n+1}-522x_{n+1})^2-2$
- $\frac{-29232x_{4n+4}+56y_{4n+5}}{28}+4\left(\frac{-29232x_{2n+2}+56y_{2n+3}+56}{28}\right)-2$
- $2y_{4n+3}+8y_{2n+1}+6$
- $\frac{3134y_{4n+4}-522x_{4n+5}}{28}+4\left(\frac{3134y_{2n+2}-522x_{2n+3}+56}{28}\right)-2$
- $(3134y_{4n+5}-29232x_{4n+5})+4(3134y_{2n+3}-29232x_{2n+3}+2)-2$
- $2y_{4n+3}+8y_{2n+1}+14$
- $\frac{336y_{4n+4}-6y_{4n+5}+1344y_{2n+2}-24y_{2n+3}+42}{3}$

➢ Employing linear combinations among the solutions , one obtains solutions to other choices of hyperbolas

**Choice 1:**

Let X= $2x_{n+1}-56x_{n,}$ , Y= $2\sqrt{87}x_n$

$$X^2-9Y^2=36$$

Note that (X,Y)satisfies the hyperbola

**Choice 2:**

Let X= $x_{n+2}-1567x_n$, Y $=2\sqrt{87}x_n$

$$X^2-7056Y^2=28224$$

Note that (X, Y) satisfies the hyperbola

**Choice 3:**

Let X= $56\sqrt{87}x_{n+1}-6y_{n+1}$, Y= $56y_{2n+2}-522x_{2n+2}$

$$Y^2-X^2=4$$

Note that (X, Y) satisfies the hyperbola

**Choice 4:**

Let X= $2y_{n+1}-536x_n$, Y= $2\sqrt{87}x_n$

$$X^2-272832\,Y^2=3136$$

Note that (X,Y) satisfies the hyperbola

**Choice 5:**

Let X= $2y_{n+2}-29232x_n$, Y= $2\sqrt{87}x_n$

$$X^2-(2455489*348)Y^2=9821956$$

Note that (X, Y) satisfies the hyperbola

**Choice 6:**

Let X= $56x_{n+2}-3134x_{n+1}$, Y $=2\sqrt{87}(56x_{n+1}-x_{n+2})$

$$X^2-9Y^2=36$$

Note that (X, Y) satisfies the hyperbola

**Choice 7:**

Let X= $2y_n$, Y= $\sqrt{87}(x_{n+1}-3y_n)$

$$196X^2-Y^2=784$$

Note that (X, Y) satisfies the hyperbola

**Choice 8:**

Let X= $(56\sqrt{87}x_{n+1}-6\sqrt{87}y_{n+1})^2$ , Y= $56y_{n+1}-522x_{n+1}$

$$Y^2-X^2=4$$

Note that (X, Y) satisfies the hyperbola

**Choice 9:**

Let X= $-29232x_{n+1}+56y_{n+2}$, Y= $3134\sqrt{87}x_{n+1}-6\sqrt{87}y_{n+2}$

$$X^2 - Y^2 = 3136$$

Note that (X, Y) satisfies the hyperbola

**Choice 10:**

Let X= $2y_n$, Y= $\sqrt{87}(x_{n+2} - 336y_n)$

$$X^2 - Y^2 = 9821956$$

Note that (X, Y) satisfies the hyperbola

**Choice 11:**

Let X= $3134y_{n+1} - 522x_{n+2}$, Y= $\sqrt{87}(56x_{n+2}) - 336y_{n+1}$

$$X^2 - Y^2 = 3136$$

Note that (X, Y) satisfies the hyperbola

**Choice 12:**

$LetX = 3134y_{n+2} - 29232x_{n+2}, \; Y = \sqrt{87}(3134x_{n+2} - 336y_{n+2})$

$$X^2 - Y^2 = 4$$

Note that (X, Y) satisfies the hyperbola

**Choice 13:**

$LetX = 2y_n, \quad Y = 2(y_{n+1} - 28y_n)$

$$783X^2 - Y^2 = 3132$$

Note that (X, Y) satisfies the hyperbola

**Choice 14:**

$LetX = 2y_n, Y = y_{n+2} - 1567y_n$

$$613872X^2 - Y^2 = 2455488$$

Note that (X, Y) satisfies the hyperbola

**Choice 15:**

$LetX = 336y_{n+1} - 6y_{n+2}, \quad Y = 56y_{n+2} - 3134y_{n+1}$

$$87X^2 - Y^2 = 3132$$

Note that (X, Y) satisfies the hyperbola

- Employing linear combinations among the solutions, one obtains solutions to other choices of parabolas

**Choice 16:**

$$Let X = 2x_{2n+2} - 56x_{2n+1} + 6, \qquad Y = 2\sqrt{87}x_n$$

$$X - 3Y^2 = 12$$

Note that (X, Y) satisfies the parabola

**Choice 17:**

$$Let X = x_{2n+3} - 1567x_{2n+1} + 168, \; Y = 2\sqrt{87}x_n$$

$$X - 84y^2 = 336$$

Note that (X, Y) satisfies the parabola

**Choice 18:**

$$\text{Let } X = 56y_{2n+2} - 522x_{2n+2}, \quad Y = 56\sqrt{87}x_{n+1} - 6y_{n+1}$$

$$Y - X^2 = 4$$

Note that (X, Y) satisfies the parabola

**Choice 19:**

$$Let Y = \frac{2y_{2n+2} - 536x_{2n+1} + 56}{28}, \; X = 2y_{n+1} - 536x_n$$

$$Y - X^2 = 4$$

Note that (X, Y) satisfies the parabola

**Choice 20:**

$$Let X = 1567(2y_{2n+3} - 29232x_{2n+1} + 3134), \; Y = 2\sqrt{87}x_n$$

$$\frac{X - 1567^2 Y^2}{(1567)^2} = 4$$

Note that (X, Y) satisfies the parabola

**Choice 21:**

$$Let X = 56x_{2n+3} - 3134x_{2n+2} + 6, \; Y = 2\sqrt{87}^2(56x_{n+11} - x_{n+2})^2$$

$$X - 3Y^2 = 12$$

Note that (X, Y) satisfies the parabola

**Choice 22:**

$$Let X = 2y_{2n+1} + 2,\ Y = \sqrt{87}\left(x_{n+1} - 3y_n\right)$$

$$196X - Y^2 = 784$$

Note that (X, Y) satisfies the parabola

**Choice 23:**

$$Let Y = 56y_{2n+2} - 522x_{2n+2}\ ,\quad X = 56\sqrt{87}x_{n+1} - 6\sqrt{87}y_{n+1}$$

$$Y - X^2 = 4$$

Note that (X, Y) satisfies the parabola

**Choice 24:**

$$Let Y = 3134\sqrt{87}X_{n+1} - 6\sqrt{87}y_{n+2},\ X = -29232x1_{n+1} + 56y_{n+2}$$

$$X + Y^2 = 3136$$

Note that (X, Y) satisfies the parabola

**Choice 25:**

$$Let Y = \sqrt{87}\left(x_{n+2} - 336y_n\right),\ X = 2y_{n+2} + 2$$

$$2455489\, X - Y^2 = 9821956$$

Note that (X, Y) satisfies the parabola

**Choice 26:**

$$Let Y = \sqrt{87}\left(56x_{n+2} - 336y_{n+1}\right),\ X = 3134y_{2n+2} - 522x_{2n+3} + 56$$

$$28X - Y^2 = 3136$$

Note that (X, Y) satisfies the parabola

**Choice 27:**

$$Let Y = \sqrt{87}\left(3134x_{n+2} - 336y_{n+2}\right),\ X = 3134y_{2n+3} - 29232x_{2n+3} + 2$$

$$X - Y^2 = 4$$

Note that (X, Y) satisfies the parabola.

**Choice 28:**

$$Let Y = y_{n+2} - 1567y_n\ ,\ and\ X = 2y_{2n+1} + 2$$

$$613872X - Y^2 = 2455488$$

Note that (X,Y) satisfies the parabola

**Choice 29:**

$$Let Y = 2(Y_{n+1} - 28y_n), \quad X = 2y_{2n+1} + 2$$

$$X - Y^2 = 3132$$

Note that (X, Y) satisfies the parabola

**Choice 30:**

$$Let Y = 2\sqrt{87}x_n, \quad X = 2y_{n+1} + 2$$

$$X - Y^2 = 4$$

Note that (x,y) satisfies the parabola

$\Rightarrow$ *Considering suitable values of* $x_n$ & $y_n$, *one generates* $2^{nd}$ order Ramanujan numbers with base integers as real integers.

For illustration, consider

$$x_1 = 168 = 2*84 = 3*56 = 4*42 = 6*28 = 7*24 = 8*21 = 12*14$$

Now,

$1*168 = 2*84$
$\Rightarrow (1+168)^2 + (2-84)^2 = (1-168)^2 + (2+84)^2 = 35285$

$1*168 = 3*56$
$\Rightarrow (1+168)^2 + (3-56)^2 = (1-168)^2 + (3+56)^2 = 31370$

$1*168 = 4*42$
$\Rightarrow (1+168)^2 + (4-42)^2 = (1-168)^2 + (4+42)^2 = 30005$

$1*168 = 6*28$
$\Rightarrow (1+168)^2 + (6-28)^2 = (1-168)^2 + (6+28)^2 = 29045$

$1*168 = 7*24$
$\Rightarrow (1+168)^2 + (7-24)^2 = (1-168)^2 + (7+24)^2 = 28850$
$1*168 = 8*21$
$\Rightarrow (1+168)^2 + (8-21)^2 = (1-168)^2 + (8+21)^2 = 28730$
$1*168 = 12*14$
$\Rightarrow (1+168)^2 + (12-14)^2 = (1-168)^2 + (12+14)^2 = 28565$

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$2*84 = 3*56$

$\Rightarrow (2+84)^2 + (3-56)^2 = (2-84)^2 + (3+56)^2 = 10205$

$2*84 = 4*42$

$\Rightarrow (2+84)^2 + (4-42)^2 = (2-84)^2 + (4+42)^2 = 8840$

$2*84 = 6*28$

$\Rightarrow (2+84)^2 + (6-28)^2 = (2-84)^2 + (6+28)^2 = 7880$

$2*84 = 7*24$

$\Rightarrow (2+84)^2 + (7-24)^2 = (2-84)^2 + (7+24)^2 = 7685$

$2*84 = 8*21$

$\Rightarrow (2+84)^2 + (8-21)^2 = (2-84)^2 + (8+21)^2 = 7565$

$2*84 = 12*14$

$\Rightarrow (2+84)^2 + (12-14)^2 = (2-84)^2 + (12+14)^2 = 7400$

$3*56 = 4*42$

$\Rightarrow (3+56)^2 + (4-42)^2 = (3-56)^2 + (4+42)^2 = 4925$

$3*56 = 6*28$

$\Rightarrow (3+56)^2 + (6-28)^2 = (3-56)^2 + (6+28)^2 = 3965$

$3*56 = 7*24$

$\Rightarrow (3+56)^2 + (7-24)^2 = (3-56)^2 + (7+24)^2 = 3770$

$3*56 = 8*21$

$\Rightarrow (3+56)^2 + (8-21)^2 = (3-56)^2 + (8+21)^2 = 3650$

$3*56 = 12*14$

$\Rightarrow (3+56)^2 + (12-14)^2 = (3-56)^2 + (12+14)^2 = 3485$

$4*42 = 6*28$

$\Rightarrow (4+42)^2 + (6-28)^2 = (4-42)^2 + (6+28)^2 = 2600$

$4*42 = 7*24$

$\Rightarrow (4+42)^2 + (7-24)^2 = (4-42)^2 + (7+24)^2 = 2405$

$4*42 = 8*21$

$\Rightarrow (4+42)^2 + (8-21)^2 = (4-42)^2 + (8+21)^2 = 2285$

$4*42 = 12*14$

$\Rightarrow (4+42)^2 + (12-14)^2 = (4-42)^2 + (12+14)^2 = 2120$

$6*28=7*24$

$\Rightarrow (6+28)^2+(7-24)^2=(6-28)^2+(7+24)^2=1445$

$6*28=8*21$

$\Rightarrow (6+28)^2+(8-21)^2=(6-28)^2+(8+21)^2=1325$

$6*28=12*14$

$\Rightarrow (6+28)^2+(12-14)^2=(6-28)^2+(12+14)^2=1160$

$7*24=8*21$

$\Rightarrow (7+24)^2+(8-21)^2=(7-24)^2+(8+21)^2=1130$

$7*24=12*14$

$\Rightarrow (7+24)^2+(12-14)^2=(7-24)^2+(12+14)^2=965$

$8*21=12*14$

$\Rightarrow (8+21)^2+(12-14)^2=(8-21)^2+(12+14)^2=845$

Thus **35285, 31370, 30005, 29045 , 28850 , 28730 , 28565 , 10205 , 8840 , 7880, 7685 , 7565 , 7400 , 4925 , 3965 , 3770 , 3650 , 3485 , 2600 , 2405 , 2285 , 2120 , 1445 , 1325 , 1160 , 1130 , 965 ,845** represent $2^{nd}$ order Ramanujan numbers.

$\Rightarrow$ Considering suitable values of $x_n$ & $y_n$ , one generates $2^{nd}$ order Ramanujan number with base integers as Gaussian integers.

For illustration, consider again

$1*168=2*84$

$\Rightarrow (1+168i)^2+(2-84i)^2=(1-168i)^2+(2+84i)^2=-35275$

$1*168=3*56$

$\Rightarrow (1+168i)^2+(3-56i)^2=(1-168i)^2+(3+56i)^2=-31350$

$1*168=4*42$

$\Rightarrow (1+168i)^2+(4-42i)^2=(1-168i)^2+(4+42i)^2=-29971$

$1*168=6*28$

$\Rightarrow (1+168i)^2+(6-28i)^2=(1-168i)^2+(6+28i)^2=-28971$

$1*168=7*24$

$\Rightarrow (1+168i)^2+(7-24i)^2=(1-168i)^2+(7+24i)^2=-28750$

$1*168=8*21$

$\Rightarrow (1+168i)^2+(8-21i)^2=(1-168i)^2+(8+21i)^2=-28600$

$1*168 = 12*14$
$\Rightarrow (1+168i)^2 + (12-14i)^2 = (1-168i)^2 + (12+14i)^2 = -28275$

$2*84 = 3*56$
$\Rightarrow (2+84i)^2 + (3-56)i^2 = (2-84i)^2 + (3+56i)^2 = -10179$

$2*84 = 4*42$
$\Rightarrow (2+84i)^2 + (4-42i)^2 = (2-84i)^2 + (4+42i)^2 = -8800$

$2*84 = 6*28$
$\Rightarrow (2+84i)^2 + (6-28i)^2 = (2-84i)^2 + (6+28i)^2 = -7800$

$2*84 = 7*24$
$\Rightarrow (2+84i)^2 + (7-24i)^2 = (2-84i)^2 + (7+24i)^2 = -7579$

$2*84 = 8*21$
$\Rightarrow (2+84i)^2 + (8-21i)^2 = (2-84i)^2 + (8+21i)^2 = -7429$

$2*84 = 12*14$
$\Rightarrow (2+84i)^2 + (12-14i)^2 = (2-84i)^2 + (12+14i)^2 = -7104$

$3*56 = 4*42$
$\Rightarrow (3+56i)^2 + (4-42i)^2 = (3-56i)^2 + (4+42i)^2 = -4875$

$3*56 = 6*28$
$\Rightarrow (3+56i)^2 + (6-28i)^2 = (3-56i)^2 + (6+28i)^2 = -3875$

$3*56 = 7*24$
$\Rightarrow (3+56i)^2 + (7-24i)^2 = (3-56i)^2 + (7+24i)^2 = -3654$

$3*56 = 8*21$
$\Rightarrow (3+56i)^2 + (8-21i)^2 = (3-56i)^2 + (8+21i)^2 = -3504$

$3*56 = 12*14$
$\Rightarrow (3+56i)^2 + (12-14i)^2 = (3-56i)^2 + (12+14i)^2 = -3179$

$4*42 = 6*28$
$\Rightarrow (4+42i)^2 + (6-28i)^2 = (4-42i)^2 + (6+28i)^2 = -2496$

$4*42 = 7*24$
$\Rightarrow (4+42i)^2 + (7-24i)^2 = (4-42i)^2 + (7+24i)^2 = -2275$

$4*42 = 8*21$
$\Rightarrow (4+42i)^2 + (8-21i)^2 = (4-42i)^2 + (8+21i)^2 = -2125$

$4*42=12*14$

$\Rightarrow (4+42i)^2+(12-14i)^2=(4-42i)^2+(12+14i)^2=-1800$

$6*28=7*24$

$\Rightarrow (6+28i)^2+(7-24i)^2=(6-28i)^2+(7+24i)^2=-1275$

$6*28=8*21$

$\Rightarrow (6+28i)^2+(8-21i)^2=(6-28i)^2+(8+21i)^2=-1125$

$6*28=12*14$

$\Rightarrow (6+28i)^2+(12-14i)^2=(6-28i)^2+(12+14i)^2=-800$

$7*24=8*21$

$\Rightarrow (7+24i)^2+(8-21i)^2=(7-24i)^2+(8+21i)^2=-904$

$7*24=12*14$

$\Rightarrow (7+24i)^2+(12-14i)^2=(7-24i)^2+(12+14i)^2=-579$

$8*21=12*14$

$\Rightarrow (8+21i)^2+(12-14i)^2=(8-21i)^2+(12+14i)^2=-429$

Note that **-35275, -31350, -29971, -28971, -28750, -28600, -28275, -10179, -8800, -7800, -7579, -7429, -7104, -4875, -3875, -3654, -3504, -3179, -2496, -2275, -2125, -1800, -1275, -1125, -800, -904, -579, -429** represent $2^{nd}$ order Ramanujan numbers with base integers as Gaussian integers.

CONCLUSION:

In this paper , an attempt has been made to obtain non-zero distinct integer solutions to the binary quadratic diophantine equation in title representing hyperbolas. As there are varieties of hyperbolas , the readers may search for other forms of hyperbolas to obtain integer solutions for the corresponding hyperbolas.

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